IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

A generalized approach for the description of magnetostatic soliton interaction in yttrium-iron-garnet films

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2000 J. Phys.: Condens. Matter 12 8875

(http://iopscience.iop.org/0953-8984/12/41/313)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.221
The article was downloaded on 16/05/2010 at 06:53

Please note that terms and conditions apply.

J. Phys.: Condens. Matter **12** (2000) 8875–8882. Printed in the UK PII: S0953-8984(00)14484-4

A generalized approach for the description of magnetostatic soliton interaction in yttrium–iron-garnet films

R Khomeriki† and L Tkeshelashvili‡

† Department of Physics, Tbilisi State University, Chavchavadze Avenue 3, Tbilisi 380028, Georgia

‡ Institute of Physics, Tamarashvili Street 6, Tbilisi 380077, Georgia

E-mail: khomeriki@hotmail.com (R Khomeriki) and lashat@mail.com (L Tkeshelashvili)

Received 5 June 2000, in final form 5 September 2000

Abstract. The nonresonant interaction of weakly nonlinear envelope magnetostatic solitons in yttrium–iron-garnet films is theoretically studied. The nonlinear coefficients characterizing the interaction between forward volume wave solitons in perpendicularly magnetized films as well as the collision of backward volume wave bright solitons with surface wave dark solitons in in-plane-magnetized films are calculated. It is shown that the manifestations of the interaction process are experimentally observable using perpendicularly propagating solitons.

1. Introduction

In many physical systems it is possible to discover localized structures (solitons, kinks, breathers, etc) [1–5]. In this connection, the observations of magnetostatic envelope solitons in yttrium–iron-garnet (YIG) thin films should be especially singled out [6–18]. The low damping in this medium allows one to observe the formation, propagation, and reflection of magnetostatic envelope solitons even at room temperature. All theoretically predicted creations [19], such as bright forward volume wave (FVW) [6–10], backward volume wave (BVW) [8–14], and dark surface wave (SW) solitons [15] were experimentally observed. Their formation and propagation are described in terms of the nonlinear Schrödinger (NLS) equation [3].

At the same time, the effects concerning the interaction of solitons in YIG films have not been sufficiently studied, apparently because of a weak theoretical base. The observations of soliton collisions were made mainly in the case of BVW solitons in in-plane-magnetized films [11]. It should be mentioned that only the interaction of single-space-dimensional envelope solitons could be considered since two-space-dimensional soliton collisions lead to their destruction [16]. In our recent articles we theoretically studied the soliton interaction in bulk magnetic samples [20] as well as in YIG thin films for carrier wavenumbers $k \gg 1/L$ (L is the film thickness) [21]. However, in such media and for the above-mentioned wavenumber range, even soliton propagation has not been experimentally observed and the studies [20, 21] were carried out to show the benefits of the method used (the reductive perturbation approach [22, 23]). Note that in the above-mentioned wavenumber range $k \gg 1/L$, YIG film also could be considered as a bulk sample as long as such carrier waves do not 'feel' sample boundaries. Thus the evolution of the nonlinear wave has to be described by the three-dimensional NLS equation (see reference [21]), and apparently it is still impossible to fulfil experimentally the

0953-8984/00/418875+08$30.00 © 2000 IOP Publishing Ltd

homogeneity criterion along the direction perpendicular to the film plane which is necessary for creation of one-dimensional envelope solitons.

Another situation arises in the case of $k \ll 1/L$ carrier wavenumbers considered in the present article. Here the boundary conditions fix the third dimension, and the wave evolution is described by the two-dimensional NLS equation. Moreover, it is possible to consider the important limit $k \to 0$ for which an analytical solution of the problem can be found.

At the same time, the magnetostatic solitons considered here (carrier wavenumber range $k \ll 1/L$) have been studied extensively, and we suggest new experiments to verify the results concerning soliton interaction obtained theoretically in the present article. In particular, the parameters characterizing the interaction process are calculated, and it is shown that they are experimentally measurable. It should be mentioned that the use of noncollinearly propagating solitons is preferable for observing the effects of interaction between them. As will be shown below, the interaction effects are nonzero even for perpendicularly propagating solitons, unlike in the previously examined case [21]. Therefore it becomes possible to see simply the interaction effects creating the solitons on neighbouring sides of a rectangular sample (see figure 1).

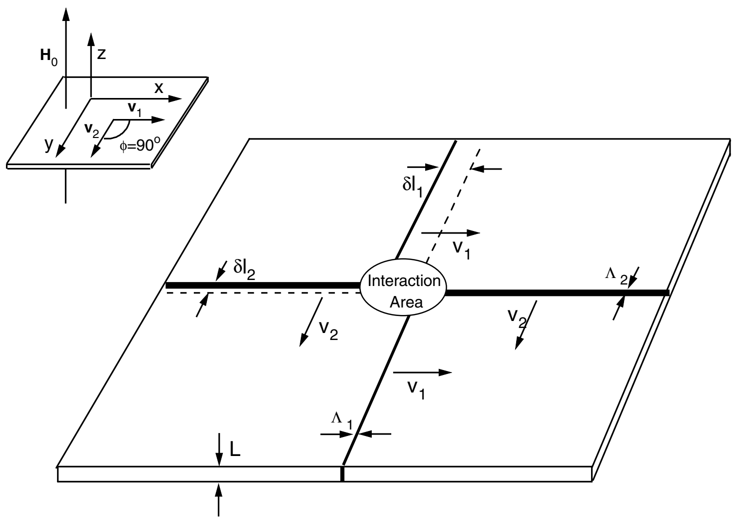

Figure 1. A schematic diagram showing the interaction of envelope solitons with group velocities v_1 and v_2 and widths Λ_1 and Λ_2. δl_1 and δl_2 are the shifts caused by the interaction, L is the disc thickness, H_0 is the internal magnetic field, ϕ is the angle between the propagation directions.

2. Interaction of FVW solitons

Firstly we consider the collision of FVW solitons in perpendicularly magnetized films. The stability of such solitons is experimentally confirmed in references [6–10]. Moreover, it was shown that FVW solitons retain their shapes and velocities after interaction [17, 18]. On the other hand, the well-known issues relating to the envelope soliton interaction, particularly the shifts of phases and group velocities, have not been measured until now.

Let us consider a perpendicularly magnetized ferromagnetic film with unpinned surface spins. Let H_0 be an internal magnetic field (z-direction); $\vec{r}$ indicates the radius vector lying in the sample plane (x, y). It is well known that due to the modulation instability, longitudinally modulated magnetostatic envelope FVW solitons can be created in the film.

Using Landau–Lifshitz and magnetostatic equations together with the corresponding boundary conditions, it is possible to investigate the nonlinear evolution of the magnetization density $\vec{M}$ and the total magnetic field $\vec{H}$ in a sample. These equations have well-known forms:

$$\frac{\mathrm{d}\vec{M}}{\mathrm{d}t} = -g[\vec{M} \times \vec{H}] \qquad \mathrm{div}(\vec{H} + 4\pi\vec{M}) = 0 \qquad \mathrm{rot}\,\vec{H} = 0. \tag{1}$$

Applying the ordinary perturbation approach, the nonlinear two-dimensional NLS equation is obtained [19] for propagation of the FVW envelope φ:

$$\mathrm{i}\left(\frac{\partial\varphi}{\partial t} + v_g\frac{\partial\varphi}{\partial x}\right) + \frac{1}{2}\omega_y''\frac{\partial^2\varphi}{\partial y^2} + \frac{1}{2}\omega_x''\frac{\partial^2\varphi}{\partial x^2} - D|\varphi|^2\varphi = 0 \tag{2}$$

where φ is defined by the expressions

$$\begin{gathered} m^+ = \varphi\mathrm{e}^{\mathrm{i}(\vec{k}\cdot\vec{r}-\omega t)} \qquad m^+ = m_x + \mathrm{i}m_y \qquad \vec{m} = \big(\vec{M} - \vec{M}_0\big)/|\vec{M}_0| \\ v_g = \left.\frac{\partial\omega}{\partial k}\right|_{k\to 0} = \frac{\omega_M L}{4} \qquad D\big|_{k\to 0} = \omega_M \\ \omega_x'' = \left.\frac{\partial^2\omega}{\partial k_x^2}\right|_{k\to 0} = -\frac{\omega_M^2 L^2}{16\omega_H}\left(1 + \frac{8\omega_H}{3\omega_M}\right) \qquad \omega_y'' = \left.\frac{\partial^2\omega}{\partial k_y^2}\right|_{k\to 0} = \frac{v_g}{k}. \end{gathered} \tag{3}$$

ω and $\vec{k}$ are the carrier wave frequency and wave vector, respectively; $\vec{k}$ is directed along the x-axis; v_g is the group velocity of the nonlinear FVW; D is a coefficient characterizing nonlinear 'self-action'; $\omega_M = 4\pi g M_0$; M_0 is the static magnetization; g is the gyromagnetic ratio; and $\omega_H = gH_0$. Let us note that the limit $k \to 0$ is considered as long as $kL \ll 1$ (in reference [21] another limit, $kL \gg 1$, was examined).

As can be easily seen from (3) (see also reference [19]), $\omega_x'' D < 0$ and $\omega_y'' D > 0$; thus nonlinear FVW are unstable against longitudinal and stable against transverse modulations. Therefore, the one-dimensional soliton solution with $\partial^2\varphi/\partial y^2 = 0$, i.e. the solution of the one-dimensional NLS equation

$$\mathrm{i}\left(\frac{\partial\varphi}{\partial t} + v_g\frac{\partial\varphi}{\partial x}\right) + \frac{1}{2}\omega_x''\frac{\partial^2\varphi}{\partial x^2} - D|\varphi|^2\varphi = 0 \tag{4}$$

is stable. The one-soliton solution of (4) has a well-known profile:

$$|\varphi| = |\varphi|_{max}\,\mathrm{sech}\left(\frac{x - v_g t}{\Lambda}\right) \qquad \Lambda = \left|\frac{\omega_x''}{D}\right|^{1/2}\frac{1}{|\varphi|_{max}}. \tag{5}$$

The NLS equation (4), as well as equation (2), describes the evolution of a weakly nonlinear FVW envelope with a definite carrier wavenumber and frequency, and direction of the propagation velocity (along the x-axis). Consequently, the N-soliton solution may represent an interaction of the envelope solitons with the same ω and k and propagating along the x-direction. Thus the interaction of solitons propagating noncollinearly and even the collision of the solitons propagating with opposite velocities [11,17,18] cannot be described by the above-mentioned description. Therefore, we use in the present article the two-dimensional extension [20,21] of the reductive perturbation approach [23] in order to study the stable FVW envelope soliton interaction propagating noncollinearly in YIG thin films. Considering the nonresonant interaction between two weakly nonlinear waves according to a well-established procedure [20,21,23], we search for the solution of (1) as follows:

$$\vec{m}(z,\vec{r},t) = \sum_{\alpha=1}^{\infty}\varepsilon^{\alpha}\sum_{l,n=-\infty}^{\infty}\vec{m}_{ln}^{(\alpha)}(z,\eta_1,\eta_2,\tau)\exp\left[\mathrm{i}(\vec{k}_{ln}\cdot\vec{r} - \omega_{ln}t) + \sum_{\beta=1}^{\infty}\varepsilon^{\beta}\Omega_{ln}^{(\beta)}(\eta_1,\eta_2,\tau)\right]. \tag{6}$$

Here $\vec{k}_{ln} = l\vec{k}_1 + n\vec{k}_2$; $\omega_{ln} = l\omega_1 + n\omega_2$; ω_1, $\vec{k}_1$ and ω_2, $\vec{k}_2$ are the carrier frequencies and wave vectors of the first and second nonlinear waves, respectively; $\Omega_{ln}^{(\beta)} = l\Omega_1^{(\beta)} + n\Omega_2^{(\beta)}$ are phase shifts induced by the interaction; η_i ($i = 1, 2$) and τ are 'slow' space-time variables [20, 21, 23]:

$$\eta_i = \varepsilon\left(\vec{\kappa}_i \cdot \vec{r} - v_{gi}t - \sum_{\gamma=0}^{\infty} \varepsilon^\gamma \psi_i^{(\gamma)}(\eta_1, \eta_2, \tau)\right) \qquad \tau = \varepsilon^2 t. \tag{7}$$

$\vec{\kappa}_i = \vec{k}_i/k_i$ (the longitudinal modulations are examined); v_{g1} and v_{g2} are the group velocities of the first and second solitons, respectively. The corrections $\Omega_{ln}^{(\beta)}$ and $\phi_i^{(\gamma)}$ are induced by the interaction and are manifested in the appearance of group velocity shifts δv_{gi} of the interacting solitons. ε is a formal small parameter connected with the smallness of the interacting solitons' amplitudes. In the final results, ε will enter in combination with the soliton amplitude $|\varphi|_{max} \equiv |m^+|_{max}$ and, as long as the weakly nonlinear solution is examined ($|m^+| \ll 1$), $|m^+|_{max}$ will play the role of a small parameter ε. In this connection it should also be noted that in the experiments on magnetization of solitons in YIG films, the value of $|M^+|$ is usually less than 10% of the static magnetization. Thus the weakly nonlinear approach examined in the present article is valid for describing almost all experimental data.

Further, it is assumed that in the major approximation only the harmonics $\varphi_1 \equiv \varphi_{10}^{(1)}$ and $\varphi_2 \equiv \varphi_{01}^{(1)}$ are nonzero. As follows from the calculations (see for details [20, 21, 23]), they are solutions of the NLS equation (4). Thus we get generally an $(N+N)$-soliton solution describing the nonresonant interaction of two solitary creations. Each of them can be considered as an N-soliton solution outside of the interaction area. The interaction effects reduce to the phase shifts and shifts of group velocities during the interaction. As was mentioned in our previous article, the latter effect (the shift of the propagation velocity) could be experimentally measured. Indeed the shift of the propagation velocity causes the shift of the interacted and still noninteracted parts of the nonlinear wave (see figure 1). That is, these quantities are most convenient for the experimental measurements. In figure 1 the shifts are indicated by δl_1 and δl_2 for the first and second nonlinear waves, respectively. For instance, the shift of the first soliton induced by the interaction with the second one can be simply presented in terms of the following relations [20, 21, 23]:

$$\begin{aligned}\delta l_1 &= \int_{-\infty}^{\infty} \delta v_{g1}\,\mathrm{d}t = \int_{-\infty}^{\infty} \frac{\partial \psi_1^{(1)}}{\partial \eta_2}\,\mathrm{d}\eta_2 = -\int_{-\infty}^{\infty} \frac{\partial}{\partial \vec{k}_1}\left(\frac{\partial \Omega_1^{(1)}}{\partial \eta_2}\right)\mathrm{d}\eta_2 \\ &= -\vec{\kappa}_1 \frac{\partial}{\partial \vec{k}_1}\left(\frac{D_1}{2(v_{g1}\cos\phi - v_{g2})}\right)\int_{-\infty}^{\infty} |\varphi_2|^2\,\mathrm{d}\eta_2\end{aligned} \tag{8}$$

where ϕ is the angle between the propagation directions of the interacting nonlinear waves; D_1 is a nonlinear coefficient characterizing the influence of the second soliton upon the first one while the interaction process is proceeding. After straightforward but rather complicated calculations (in full analogy with references [20, 21]), we obtain the following expressions for D_1:

$$D_1|_{k_1,k_2\to 0} = -4\omega_M \qquad \vec{\kappa}_1 \frac{\partial D_1}{\partial \vec{k}_1}\bigg|_{k_1,k_2\to 0} = \frac{7}{6}\omega_M L. \tag{9}$$

For estimates we consider now the 1 + 1 solution, i.e. the interaction of two solitons with sech-type profiles [3] (φ_1 and φ_2 are one-soliton solutions of the NLS equation (2)). Then, in view of the following relations:

$$\vec{\kappa}_1 \frac{\partial v_{g1}}{\partial \vec{k}_1}\bigg|_{k_1\to 0} = \frac{\partial^2 \omega_1}{\partial k_1^2}\bigg|_{k_1\to 0} = -\frac{\omega_M L^2}{16\omega_H}\left(1 + \frac{8\omega_H}{3\omega_M}\right) \qquad \frac{\partial v_{g2}}{\partial \vec{k}_1} = 0$$

expression (8) can be rewritten as follows:

$$\frac{\delta l_1}{\Lambda_1} = \frac{|\varphi_1|_{max}|\varphi_2|_{max}}{(1-\cos\phi)^2}\left[\frac{14}{3} + \left(6 + 4\frac{\omega_M}{\omega_H}\right)\cos\phi\right] \tag{10}$$

where Λ_1 is the width of the first soliton.

Let us take the following parameters as estimates: $|\varphi_1|_{max} = |\varphi_2|_{max} = 0.1$; $H_M \equiv \omega_M/g = 1750$ Oe; $L = 10$ μm, and $H_0 = 580$ Oe. Then the width of the one-soliton solution takes the value $\Lambda_1 \simeq 0.06$ cm and the propagation velocities are equal to $v_{g1} \simeq v_{g2} \simeq 7.5 \times 10^7$ cm s^{-1}.

We have two restrictions on the values of the parameters of the problem:

(1) The internal static magnetic field should not be less than $0.3\omega_M/g$. Otherwise the threshold for three-magnon processes is reached and nonlinear wave creation will decay rapidly.
(2) As long as only the nonresonant interaction is considered, the following inequality should be satisfied [23]: $1 - \cos\phi \gg \varepsilon$ (ε is of the order of the pulse amplitudes $|\varphi_1|_{max}$ and $|\varphi_2|_{max}$).

The angle dependence for the relative shift of the wave front $\delta l_1/\Lambda_1$ is presented in figure 2. As we see, in the case where $\cos\phi = -1$ (i.e. interaction of solitons with opposite velocities), the relative shift is small: $\delta l_1/\Lambda_1 \simeq -0.03$. Besides this, it could not be directly seen and the comparison of two experiments with and without the interaction is necessary (measuring the corresponding delay time $\delta t_1 = \delta l_1/v_{g1} \simeq -0.025$ ns). Therefore it is doubtful whether one could see the manifestations of effects induced by the interaction with the second soliton. On the other hand, e.g. for $\cos\phi = 0.4$, the relative shift $\delta l_1/\Lambda_1 \simeq 0.3$; thus the shift is almost commensurate with the soliton width and it can be directly observed experimentally.

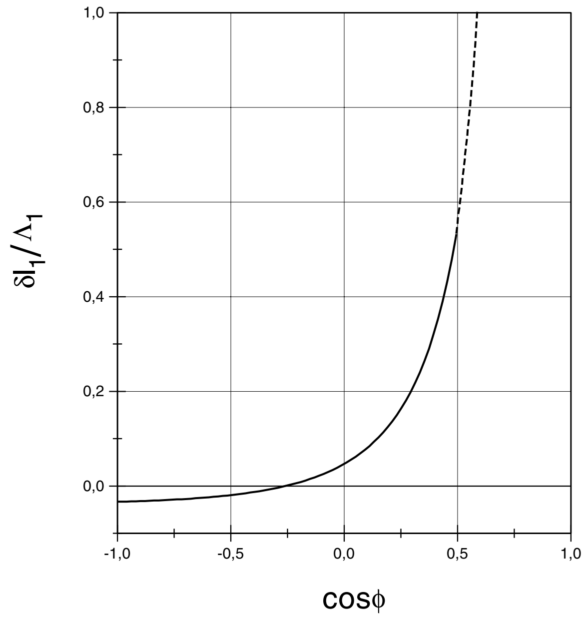

Figure 2. The angular dependence of the ratio $\delta l_1/\Lambda_1$. δl_1 is the shift of the first soliton caused by the interaction, Λ_1 is the width of the soliton, ϕ is the angle between the group velocities of the interacting solitons. The dashed line indicates the range of ϕ where the condition of nonresonant character of the interaction is violated.

The effect could be significantly increased by examining the $(1 + N)$-soliton solution, i.e. the influence of the N-soliton impulse with envelope φ_2 upon the one-soliton impulse with envelope φ_1. In this case the corresponding shift of the wave front of the first soliton will increase at least N times (the case of N independent one-soliton impulses) while the pulse power required for the creation of an N-soliton impulse should be $(2N - 1)^2$ times greater [3, 19]. It should be mentioned that if both nonlinear waves are one-soliton pulses, then $\Lambda_1 \simeq \Lambda_2$, while if the first and second nonlinear waves are one- and N-soliton impulses, respectively, Λ_2 is much larger than Λ_1 in the case where $|\varphi_1|_{max} \sim |\varphi_2|_{max}$. Consequently, the shift of the wave front of the first soliton δl_1 will be larger than the shift δl_2.

In contrast to the case considered in reference [21], the shifts δl_1 and δl_2 are nonzero for perpendicularly propagating solitons, as follows from expression (10). In particular, for the 1 + 1 solution and the above-chosen parameters, $\delta l_1/\Lambda_1 \simeq \delta l_2/\Lambda_2 \simeq 0.05$.

3. Interaction between bright and dark solitons

Let us consider now a most interesting case of interaction between different types of soliton, particularly the collision between bright and dark solitons. For that purpose let us examine the interaction of a nonlinear spin wave with carrier frequency and wavenumber ω_1, $\vec{k}_1$ with a second one with the parameters ω_2 and $\vec{k}_2$ in in-plane-magnetized YIG film with unpinned surface spins. Let us suppose that $\vec{k}_1$ is parallel (BVW case) to the static magnetic field lying in the film plane along the x-direction and that $\vec{k}_2$ is perpendicular (y-direction) to the static field (SW case). The propagation of such nonlinear waves out of the interaction area could be described in terms of the same two-dimensional NLS equation (2) with the nonlinear coefficient of 'self-action' being the same for both BVW and SW nonlinear waves [19]:

$$D|_{\vec{k}\to 0} = -\frac{\omega_M\omega_H}{4\omega_0} \qquad \omega_0 = \sqrt{\omega_H(\omega_H + \omega_M)}. \tag{11}$$

We can also write down the expressions for propagation velocities and dispersion coefficients:

$$\begin{aligned} v_{gB} &= -\frac{\omega_H\omega_M}{4\omega_0}L \\ \omega''_{xB} &= \frac{5}{48}\frac{\omega_M\omega_H}{\omega_0}\left(1 + \frac{3\omega_H^2}{5\omega_0^2}\right)L^2 \qquad \omega''_{yB} = \frac{\omega_M^2 L}{4\omega_0 k_1}\left(2 + \frac{\omega_H}{\omega_M}\right) \end{aligned} \tag{12}$$

for the BVW case and

$$\begin{aligned} v_{gS} &= \frac{\omega_M^2 L}{4\omega_0} \\ \omega''_{xS} &= -\frac{\omega_M^2 L}{4\omega_0 k_2}\left(1 + 2\frac{\omega_H}{\omega_M}\right) \qquad \omega''_{yS} = -\frac{\omega_M^4 L^2}{16\omega_0^3}\left(1 + 8\frac{\omega_0^2}{\omega_M^2}\right) \end{aligned} \tag{13}$$

for the SW case†. As can be easily seen from expressions (11)–(13), the following inequalities hold for the BVW case: $D\omega''_{yB} > 0$ and $D\omega''_{xB} > 0$. This means that the BVW plane waves are unstable against both longitudinal and transverse modulations and longitudinally modulated BVW bright solitons could be formed [8–14] with a profile

$$|\varphi_1| = |\varphi_1|_{max}\,\mathrm{sech}\left(\frac{x - v_{gB}t}{\Lambda_1}\right) \qquad \Lambda_1 = \left|\frac{\omega''_{xB}}{D}\right|^{1/2}\frac{1}{|\varphi_1|_{max}}. \tag{14}$$

† We mention that for the SW case in the NLS equation (2) the term $v_g\,\partial\varphi/\partial x$ should be replaced by $v_{gS}\,\partial\varphi_2/\partial y$ because the carrier frequency of the SW is taken to be directed along $\vec{y}$ and therefore the propagation velocity $\vec{v}_{gS}$ lies along the same axis.

Another situation arises for the SW case for which $D\omega''_{xS} < 0$ and $D\omega''_{yS} < 0$. As had been predicted theoretically (see e.g. [2]) and observed experimentally [15], the dark solitons could be created in this case with a profile as follows:

$$|\varphi_2| = |\varphi_2|_{max} \left| \frac{\sqrt{1-A^2}}{A} + \mathrm{i}\tanh\left(\frac{y - v_{gS}t}{\Lambda_2}\right)\right| \qquad \Lambda_2 = \left|\frac{\omega''_{yS}}{D}\right|^{1/2} \frac{1}{|\varphi_2|_{max}} \tag{15}$$

where A denotes the contrast of the soliton. Taking A equal to 1 (see reference [15]), we can rewrite $|\varphi_2|^2$ as the sum of the background and a sech^2-type inverse moving bump:

$$|\varphi_2|^2 = |\varphi_2|^2_{max} - |\varphi_2|^2_{max}\,\mathrm{sech}^2\left(\frac{y - v_{gS}t}{\Lambda_2}\right). \tag{16}$$

Applying again the general approach [20, 21] for calculation of the nonlinear coefficient characterizing the nonresonant interaction between the nonlinear BVW and SW, the following expression was obtained:

$$\vec{\kappa}_1 \frac{\partial D_1}{\partial \vec{k}_1}\bigg|_{k_1, k_2 \to 0} = \omega_M L. \tag{17}$$

Note that as long as the angle ϕ between the propagation velocities of the interacting BVW and SW solitons is equal to 90°, only the nonlinear coefficient (17) is needed to calculate the shift of the propagation velocities induced by the interaction, as follows from expression (8). Then using (8), (16), and (17), one can simply derive the formula for the delay time Δt_1 (in comparison with the time when the BVW propagates in the absence of the SW) of the BVW bright soliton:

$$\frac{\Delta t_1}{t_1} = 4\frac{\omega_0}{\omega_H}|\varphi_2|^2_{max} \tag{18}$$

(t_1 is the propagation time of the BVW soliton without interaction) and also the expression for the relative shift of the wave front (see figure 1) could be obtained:

$$\frac{\delta l_1}{\Lambda_1} = -8\frac{\omega_0}{\omega_H}\frac{\Lambda_2}{\Lambda_1}|\varphi_2|^2_{max}. \tag{19}$$

At the same time, the overall delay for the SW dark soliton is equal to zero (as long as the BVW bright soliton has zero background) and the shift of wave front is represented by the same expression (19).

For the above-chosen values of the static and demagnetizing field, soliton amplitudes (see section 2), and sample length along the x-direction $d = 1$ cm, the propagation velocities take the following values: $v_{gB} = 3.8 \times 10^7$ cm s^{-1} and $v_{gS} = 1.1 \times 10^8$ cm s^{-1}. One can also calculate the propagation time for the BVW soliton, $t_1 = 28$ ns, and the overall delay time of the BVW soliton induced by the nonlinear interaction with the SW dark soliton, $\Delta t_1 = 2.3$ ns. The latter can be compared with the BVW soliton pulse duration $\tau_0 = 0.1$ ns and it can be concluded that the delay time will be easily observed experimentally.

One can also estimate the relative shift of the wave front of the BVW bright soliton according to formula (19). For the above-chosen parameters, $\delta l_1/\Lambda_1 \simeq 0.6$. Thus it could also be observed experimentally.

4. Conclusions

A generalized approach for describing the interaction between noncollinearly propagating nonlinear magnetostatic waves in YIG films is presented. The interaction between FVW solitons in perpendicularly magnetized films as well as the collision between bright BVW solitons and dark SW solitons in in-plane-magnetized film are examined. It is also possible to apply the present approach in order to describe the interaction of nonlinear waves in other combinations. In particular, we can consider the interaction between: BVW solitons; SW dark solitons; a BVW soliton with a BVW beam (a transversely modulated soliton); a BVW bright beam with a SW dark beam (if the latter could be experimentally observed); etc.

Acknowledgments

The paper was prepared in memory of Nika and Marika Giorgadze.

References

[1] Russell J S 1840 *Trans. R. Soc. Edinburgh* **14** 47
[2] Hasegawa A 1990 *Optical Solitons in Fibres* (Berlin: Springer)
[3] Ablowitz M I and Segur H 1981 *Solitons and Inverse Scattering Transform* (Philadelphia, PA: SIAM)
[4] Moon K and Mullen K 1998 *Phys. Rev.* B **57** 1378
[5] Lepri S, Livi R and Politi A 1997 *Phys. Rev. Lett.* **78** 1896
[6] Kalinikos B A, Kovshikov N G and Slavin A N 1988 *Zh. Eksp. Teor. Fiz.* **94** 159
[7] De Gasperis P, Marcelli R and Miccoli G 1987 *Phys. Rev. Lett.* **59** 481
[8] Chen M, Tsankov M A, Nash J M and Patton C E 1994 *Phys. Rev.* B **49** 12 773
[9] Kalinikos B A, Kovshikov N G and Slavin A N 1983 *Pis. Zh. Eksp. Teor. Fiz.* **38** 343
[10] Kalinikos B A, Kovshikov N G and Slavin A N 1990 *Phys. Rev.* B **42** 8658
[11] Kovshikov N G, Kalinikos B A, Patton C E, Wright E S and Nash J M 1996 *Phys. Rev.* B **54** 15 210
[12] Boyle J W, Nikitov S A, Boardman A D, Booth J G and Booth K 1996 *Phys. Rev.* B **53** 12 173
[13] Bauer M, Büttner O, Demokritov S O, Hillebrands B, Grimalsky V, Rapoport Yu and Slavin A N 1998 *Phys. Rev. Lett.* **81** 3769
[14] Xia H, Kabos P, Patton C E and Ensle H E 1997 *Phys. Rev.* B **55** 15 018
[15] Chen M, Tsankov M A, Nash J M and Patton C E 1993 *Phys. Rev. Lett.* **70** 1707
[16] Büttner O, Bauer M, Demokritov S O, Hillebrands B, Kostilev M P, Kalinikos B A and Slavin A N 1999 *Phys. Rev. Lett.* **82** 4320
[17] Tsankov M A, Chen M and Patton C E 1994 *J. Appl. Phys.* **76** 4274
[18] Kalinikos B A and Kovshikov N G 1994 *Pis. Zh. Eksp. Teor. Phys.* **60** 290
[19] Zvezdin A K and Popkov A F 1983 *Zh. Eksp. Teor. Phys.* **84** 606
[20] Giorgadze N and Khomeriki R 1998 *J. Magn. Magn. Mater.* **186** 239
[21] Giorgadze N and Khomeriki R 1999 *Phys. Rev.* B **60** 1247
[22] Taniuti T 1974 *Prog. Theor. Phys. (Suppl.)* **55** 1
[23] Oikawa M and Yajima N 1974 *J. Phys. Soc. Japan* **37** 486